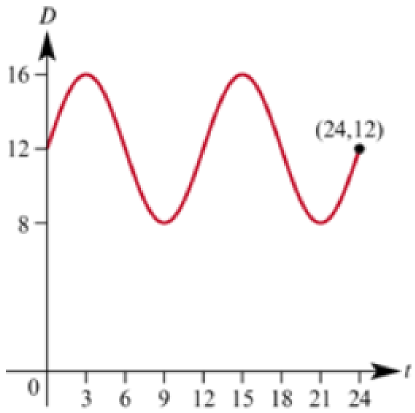
Exercise 10G: Solutions

1 a i 0.00 hours

ii 24.00 hours

b 13 February ($t = 1.48$), 24 October ($t = 9.86$)

2 a



b $t \in [0, 6] \cup [12, 18]$

c 15.9 m

3 a This occurs when $\sin 2\pi t = 1$

$x = 4 + 3 = 7\text{m}$

b This occurs when $\sin 3t = -1$

$x = 4 - 3 = 1\text{m}$

c $\sin 2\pi t = 1$ for $0 \le 2\pi t \le 4\pi$

$$2\pi t = \frac{\pi}{2}, \frac{5\pi}{2}$$
$$t = \frac{1}{4}, \frac{5}{4}$$

d $\sin 2\pi t = -1$ for $0 \le 2\pi t \le 4\pi$

$$2\pi t = \frac{3\pi}{2}, \frac{7\pi}{2}$$
$$t = \frac{3}{4}, \frac{7}{4}$$

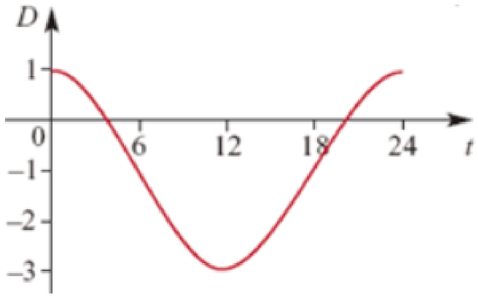
e Particle oscillates between $x = 1$ and $x = 7$

4 a $t = 4$

$$A^\circ\text{C} = 21 - 3\cos\frac{\pi}{3}$$
$$= 19.5^\circ\text{C}$$

b $$D = A - B$$
$$= -1 + 2\cos\frac{\pi t}{12}$$

c The graph is of $y = -1 + 2\cos\frac{\pi t}{12}$. It has amplitude 2, period 24, and is the cosine curve moved 1 unit down.



d $$-1+2\cos\frac{\pi t}{12}<0$$
$$\cos\frac{\pi t}{12}<\frac{1}{2}$$
$$\frac{\pi}{3}<\frac{\pi t}{12}<\frac{5\pi}{3}$$
$$4<t<20$$

From 4 am to 8 pm.

5 **a** $$\frac{\pi}{6}t-\frac{\pi}{3}=0$$
$$\frac{\pi}{6}t=\frac{\pi}{3}$$
$$t=2\text{ am}$$

b $$6+4\cos\left(\frac{\pi}{6}t-\frac{\pi}{3}\right)=2$$
$$\cos\left(\frac{\pi}{6}-\frac{\pi}{3}\right)=-1$$
$$\frac{\pi}{6}t-\frac{\pi}{3}=\pi,3\,\pi$$
$$t-2=6,18$$
$$t=8\text{ or }t=20$$

8 am and 8 pm

6 **a** **i** Amplitude $=\dfrac{5-2}{2}=1.5$ m

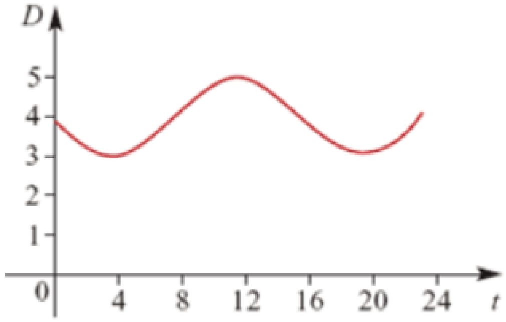
ii Period $=12$ hours

iii From graph, the shape is a cosine curve reflected in the x-axis. Graph willbe of the form
$$d(t)=-1.5\cos kt+3.5$$
$$\text{Period}=\frac{2\pi}{k}=12$$
$$k=\frac{2\pi}{12}=\frac{\pi}{6}$$
$$d(t)=-1.5\cos\frac{\pi t}{6}+3.5$$

iv 1.5 m

b 3.5 is the middle of the hour hand's path. From the graph, the distance is less than 3.5 m from the ceiling 9 am and 3 pm and between 9 pm and 3 am each day.

7 a Use the information given, starting at noon.



b In this case, it is easiest to make $t = 0$ at noon, which is the reference point.

$$D = 4 + \cos kt$$

$$\text{Period} = \frac{2\pi}{k} = 16$$

$$k = \frac{2\pi}{16} = \frac{\pi}{8}$$

$$D = 4 + \cos\frac{\pi t}{8}$$

$$D = 4 \Rightarrow \cos\frac{\pi t}{8} = 0$$

$$\frac{\pi t}{8} = \frac{\pi}{2} \text{ or } -\frac{\pi}{2}$$

$$t = 4 \text{ or } -4$$

It can enter after 8 am and must leave by 4 pm.

c

$$D = 4 + \cos\frac{\pi t}{8}$$

$$d = 3.5 \Rightarrow \cos\frac{\pi t}{8} = -0.5$$

$$\frac{\pi t}{8} = \frac{2\pi}{3} \text{ or } -\frac{2\pi}{3}$$

$$t = \frac{16}{3} \text{ or } -\frac{16}{3}$$

$$= 5\frac{1}{3} \text{ or } -5\frac{1}{3}$$

It can enter after $6{:}40$ am and must leave by $5{:}20$ pm.

8 a i

$$\frac{2\pi}{\frac{\pi}{26}} = 2\pi \times \frac{26}{\pi}$$

$$= 52 \text{ weeks } (1 \text{ year})$$

ii 3000

iii $4000 \pm 3000 = [1000, 7000]$

b i

$$N(0) = 3000\sin\frac{-10\pi}{26} + 4000$$

$$= 1194.95$$

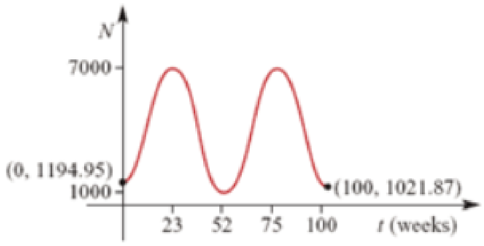
(1195 ants, more or less)

$$N(100) = 3000\sin\frac{90\pi}{26} + 4000$$

$$= 1021.87$$

(1022 ants, more or less)

ii



c **i** $3000\sin\dfrac{\pi(t-10)}{26}+4000=7000$

$\sin\dfrac{\pi(t-10)}{26}=1$

$\dfrac{\pi(t-10)}{26}=\dfrac{\pi}{2}$

$t=13+10=23$

$t=23$ and $t=75$, since the period is 52 weeks.

ii $3000\sin\dfrac{\pi(t-10)}{26}+4000=1000$

$\sin\dfrac{\pi(t-10)}{26}=-1$

$\dfrac{\pi(t-10)}{26}=\dfrac{3\pi}{2}$

$t=39+10=49$

This is the only value since the period is 52 weeks.

d $3000\sin\dfrac{\pi(t-10)}{26}+4000>5500$

$\sin\dfrac{\pi(t-10)}{26}>\dfrac{1}{2}$

$\dfrac{\pi}{6}<\dfrac{\pi(t-10)}{26}<\dfrac{5\pi}{6}$ and

$\dfrac{13\pi}{6}<\dfrac{\pi(t-10)}{26}<\dfrac{17\pi}{6}$

$\dfrac{13}{3}<t-10<\dfrac{65}{3}$ and

$\dfrac{169}{3}<t-10<\dfrac{221}{3}$

$\left(14\,\dfrac{1}{3},31\,\dfrac{2}{3}\right)\cup\left(66\dfrac{1}{3},83\dfrac{2}{3}\right)$

e The given population varies between $10\,000$ and $40\,000$, $a=15\,000$ and $d=25\,000$.
Maximum to minimum is half a period, so the period $=20$.

$2\pi\div\dfrac{\pi}{b}=20$

$2\pi=\dfrac{20\,\pi}{b}$

$b=\dfrac{20\,\pi}{2\pi}=10$

Maximum at $t=10$ means

$\dfrac{\pi\,(10-c)}{10}=\dfrac{\pi}{2}$

$10-c=5$

$c=5$